STELLAR MOTIONS IN THE ORION NEBULA CLUSTER

By

FREDERICK W. FALLON

A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA
1975

ACKNOWLEDGEMENTS

The following individuals deserve special thanks for their roles in making the present investigation possible.

For the use of observational material. Dr. K. Aa. Strand, for making available his measurements of the Yerkes photography--the more deeply appreciated in light of his own prior publication based on that material; W. D. Cannell, for the free use of his measurements of the McCormick series of 44 plates, forming the basis of his own as yet unpublished paper; N. E. Wagmann, for the free use of both early and recent plates from the Allegheny collection; P. M. Routly of the USNO for permission to use both the "SAMM" automatic and fringe-counting manual measuring engines; A. Behall and J. Josties of USNO for their help and courtesy while I was using those instruments; W. D. Heintz, for securing several recent epoch plates at the Sproul refractor; and P. Ianna for permitting me to inspect the McCormick plate collection and providing useful information concerning it.

For assistance in observing: Special thanks are due to M. DeGeorge, then Night Assistant at the USF Observatory, for assisting in the tedious--and backbreaking--work of

collimation and polar alignment of the 26" Tinsley, as well as in securing some of the photography. Graduate students L. Twigg and B. Holman also assisted in the photography, and in supporting work at the observatory generally. R. Fox helped measure and reduce the photographic photometry.

For discussions: Over the last several years Dr. H. K. Eichhorn has been a focus of inspiration and stimulating exchange of ideas. The modern era of rigorous astrometry, to which the present work belongs, owes its inception largely to his pioneering efforts. During the present work, we have discussed some of the knottier and discouraging problems that arose. I am grateful too for his critical reading of the manuscript. The statements made herein, however, except as otherwise credited, are the responsibility of the author.

Mr. A. E. Rust has given much valuable advice on computer problems, especially the optimum use of disk storage which is a crucial part of the program with which the calculations were done.

Finally, it is a great pleasure to thank Dr. S. Sofia for pointing out the need for the investigation, and for the guidance and encouragement he has given as advisor. His insight into the basic fundamentals of the problem has been invaluable, and discussions with him have always been immensely fruitful.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS ii

ABSTRACT vi

SECTION

I. INTRODUCTION 1

Background 1
The Value of a New Astrometric Study 8
Scope of Present Investigation 9

II. REFERENCE MATERIAL 11

III. THE PLATE MATERIAL 22

General Survey 22
The USF Photography 23
Coverage 26
Exposures 26
Measuring the Plates 29
Preliminary Processing 34

IV. FORMULATION OF THE PROBLEM 38

Differential Plate Measurements 38
Reference Proper Motions 41
Differential Overlap 45
Plate Overlap Method 46

V. THE COMPUTATIONS 53

Partition of the Plate Material and Treatment of the Proper Motion 53
Behavior of the Solution 55
The Computer Program 59

VI. RESULTS 63

Absolute Proper Motion of Trapezium Cluster . 63
Velocity Dispersion 64
Contraction 65

APPENDIX 73
LIST OF REFERENCES 77
BIOGRAPHICAL SKETCH 80

Abstract of Dissertation
Presented to the Graduate Council
of the University of Florida
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy

STELLAR MOTIONS IN THE ORION NEBULA CLUSTER

By

Frederick W. Fallon

December, 1975

Chairman: Sabatino Sofia
Co-Chairman: Heinrich K. Eichhorn-von Wurmb
Major Department: Astronomy

Positions and proper motions for 113 stars in the Orion Nebula are derived from measurements on 92 plates taken with a variety of telescopes, with an average epoch difference of 50 years. The positions and motions are measured with respect to a set of some 40 reference stars; they are therefore instrumentally absolute and unaffected by changes in plate scale over time. In order to incorporate a sufficient number of reference positions into the solution and to obtain the greatest possible accuracy from the data, the reductions were carried out by the plate overlap method. A new version of the method, particularly economical for the astrometry of star clusters, was employed.

It is found that 1) the velocity dispersion of the stars in the Trapezium Cluster (within 6' of the Trapezium) is less than 2½ km/sec, and 2) the Orion Nebula Cluster (radius 20')

is contracting at a linear rate of ($0\overset{''}{.}009$/yr)/degree, corresponding to 10 km/sec at the 4 parsec radius of the radio molecular cloud. These results imply that the Orion Nebula is free from large turbulent motion, is gravitationally contracting, and that star formation there has probably been proceeding continuously since it began, with the greater period of star formation yet to come. These conclusions in turn are in agreement with the large ($\sim 10^5$ solar masses) mass now assigned to the Orion Nebula.

SECTION I
INTRODUCTION

Background

The Orion Nebula has been an object of special fascination to astronomers from its discovery three centuries ago up to the present. It is now recognized as a complex involving a ("the") trapezium, a star cluster, emission nebulosity, infra-red sources and radio molecular clouds. The coincidence of these components in a single, relatively nearby (~500 pcs) object potentially makes it a key to understanding a number of basic astrophysical processes. At the same time, there are some perplexing problems which stand in the way of arriving at a unified model of the object.

Let us examine some of the special features of the Orion Nebula and the questions relating to them.

1. The Trapezium itself is an interesting object, the prototype of a small class of systems ("Trapezium system") first recognized by Ambartsumian (1955). These systems are intermediate between binaries and true clusters. Even when a trapezium is considered as part of a surrounding cluster, it is clear we are dealing with a distinct morphological group since the trapezium always stands out as a small (4-8

members), central grouping at least five magnitudes brighter than the next brighter cluster members, in contrast to ordinary open clusters. Trapezia are known to be young because they occur only among O and B stars. They would then be disrupted within about 2 to 3.10^{6} years, yet astrometric studies fail to decide whether the Orion Trapezium has positive or negative energy.

According to current thinking on the formation of trapezia, they should consist exclusively of O and B stars, yet the Orion Trapezium has two "comites" (θ'E and θ'F) both three magnitudes fainter than the faintest of the four principal members.

2. The Trapezium is surrounded by a faint cluster, the "Trapezium Cluster" (or "Orion Nebula Cluster"), discovered only as late as 1931 (Trumpler, 1931; Baade & Minkowski, 1931). Its brightest members are 11-th mag. and it is usually observed by the surrounding nebulosity. The cluster shows well in the plate reproduced in Figure 1; on the original plate the nebulosity was deliberately suppressed.

There are widely differing values given for the diameter of this cluster. Baade and Minkowski judged its diameter as 3' to 5', encompassing some 100 stars. Sharpless (1966), referring to the Baade and Minkowski discovery, misquotes the diameter as "10'"; actually, the 10' figure appears in another context in another paper by the same authors immediately preceding (in the journal) the cluster

FIGURE 1
THE ORION NEBULA SHOWING THE TRAPEZIUM CLUSTER

Two-fold enlargement of plate taken on 103a-G emulsion with 3-69 filter, exposed for 12 min. at University of South Florida Observatory. A circle 1' in radius has been drawn around the star $\theta^1 C$. The faintest stars visible are 14-th magnitude.

discovery paper. Trumpler, on the other hand, though counting the stars only in 4' diameter circle, actually gives its diameter as 15' - partly on morphological grounds. Gradually, this larger value has crept into the literature. Strand (1958) takes the diameter as 30'.

Turning to the evidence, one can see in Fig. 1 a definite concentration of stars within a circle of 2' diameter immediately surrounding the Trapezium. We shall term this group the "Trapezium cluster." It is clearly much smaller than a typical open cluster, and morphologically quite different; perhaps it might be better named the "Extended Trapezium." A star count reveals a further concentration of stars in an area of about 5' radius, extending north and also south-east toward the group θ^2. It is to be noted that heavy absorption obscures the region south-east of the Trapezium. In the absence of the dust we should expect to see cluster members in that quadrant as well. (It is to be expected too that many of the few stars visible in that area, extending for at least 20', are foreground objects.) Accordingly, we assign a diameter of 12' to this grouping, calling it the "inner Orion Nebula Cluster." Finally, in accord with current usage, we apply the name "Orion Nebula Cluster" to the whole region of ~30' in diameter centered on the Trapezium, realizing that the stars in this larger region may not comprise as well demarcated a group.

3. Separate theoretical arguments by Menon (1963, Kahn and Menon 1961) and by Vandervoort (1963), based on the mass motions and ionization of the inner region of the nebula, both lead to an age of only $2 \cdot 10^4$ years (the "ultra-short" age) for the Trapezium stars. This age refers to the time since the stars have been capable of ionizing the hydrogen.

4. The proper motions of the cluster stars can in principle yield the cluster's age, if the motions show a radial expansion. Then the kinematic age is the reciprocal of the expansion constant. (Strictly, this age refers only to the time since greatest concentration of the cluster and does not preclude earlier contraction and star formation.)

Some astrometric studies (Parenago 1953; Franz unpublished, quoted by Sharpless 1966) have indeed confirmed the ultra-short age of the Trapezium and this figure (10^4 yrs) has become widely quoted in the literature. But other astrometric studies (Strand 1958, Duboshin et al 1971) result in the much longer (though still very young) kinematic age of $2 \cdot 10^5$ years; while yet another (Meurers 1963) shows no evidence of contraction, and still others (Cannell unpublished, Vaerewyck 1972) report a slight contraction. Adding to the confusion, some (Meurers, Strand) of these studies show a <u>rotation</u> of the cluster. The compendious work of Parenago (1954) has not yet been analyzed for evidence of cluster expansion. Parenago's material is extremely heterogeneous; significantly, the stars for which he quotes the smallest proper motion errors also have the smallest dispersions in proper motions.

5. Color-magnitude diagrams for the Orion Nebula Cluster (including θ^1 and θ^2) and surrounding regions as given by several investigators, while disagreeing among themselves, agree in pointing to a young age for the stars in Orion. Some of the discrepancy can be attributed to the different sizes of the area surveyed, while some is also due to the differing reddening corrections applied. Walker (1969) presents a particularly careful study. His photometry shows the zero-age main sequence down to an intrinsic (B-V) = -.07, corresponding to a contraction time of $\sim 2\ 2/3 \cdot 10^6$ years. At fainter magnitudes, the scatter of points above the main sequence would seem to suggest non-coevality of star formation, but some of the scatter is no doubt due to the patchy distribution of the absorption.

Significantly, no photometry has been carried out on the stars of the Trapezium Cluster (except, of course, for the Trapezium itself), due to the difficulty of observing through the nebulosity. Its color-magnitude diagram might well differ from that of the larger "Nebula" Cluster, and be characteristic of an even younger age.

In this connection we should note that the unseen companion in eclipsing binary BM (=θ'B) Orionis, hitherto a very puzzling object and "black hole" candidate, has recently been explained by Popper (1975) on the basis of his discovery of its spectrum. It is apparently a $1.8M_\odot$ star in a state of pre-main sequence differential rotation,

2 magnitudes above the main sequence. The currently accepted contraction time for such an object is ~$1.8 \cdot 10^6$ years.

6. Radio observations of the molecular cloud Orion A have recently produced two surprises. First, the mass of material (mostly in the form of H_2 and not observed directly but deduced from the observed abundance of CO and HCN) is enormously large: about $10^5 M_\odot$ (Liszt et al , 1974), or <u>several hundred</u> times that of the stellar mass in the same volume. If the nebula really is expanding, it is puzzling why so large a fraction of the material failed to form into stars.

Second, the widths of the CO and HCN lines are much broader than the thermal Doppler widths, implying variable mass motion along the line of sight. Fitting a non-LTE model to the data, and assuming the motions to be large scale, Gerola and Sofia (1975) are led to conclude a linear <u>contraction</u> amounting to 12 km/sec at a distance of 1/2° from the center. This is in agreement with the large gaseous mass of the cloud and apparent range of stellar ages but not with the idea that the cluster is expanding. One may indeed interpret the line profiles by an equal <u>expansion</u> velocity, but then one finds a shell midway in the cloud where the gravitational potential energy exceeds the kinetic energy.

On the other hand, Zuckerman and Evans (1974) have criticized the uniform contraction model, proposing instead that the observed velocity spread is due to turbulent eddies interspersed all along the line of sight through the cloud. If this is so, then the cloud model of Gerola and Sofia is invalid, and some entirely different model, as yet not worked out, would be required.

The Value of a New Astrometric Study

The questions fundamentally at issue are these: (1) Is the Orion Nebula cluster expanding, as is the Orion Association as a whole, or is it still undergoing its initial contraction, with star formation still taking place? (2) Are the motions in the nebula smoothly varying, or dominated by turbulence? The answers to these questions are important to our understanding of the process of star formation in clusters.

A new study of the kinematics of the stars in the Orion Nebula would form a valuable basis for answering these questions. Thus far, the astrometric results have been discordant and inconclusive. It is the purpose of this investigation to derive proper motions of sufficiently high accuracy to make a more conclusive kinematic study possible.

Scope of Present Investigation

Four principal desiderata determined the scope of the present study. (1) The proper motions should be referred to reference stars exclusively and in no way depend on assumptions about the optical properties of the instruments with which they are obtained. We call motions of this type "instrumentally absolute." In particular, they are to be completely independent of any assumption concerning the variation of the telescope's plate scale with time. The proper motions are, of course, relative in the astrometric sense in that they are referred to the positions and motions of a system constituted by reference stars rather than referred to the equator and equinox directly. Within the accuracy of the determination, the positions and proper motions derived here may differ from the best derivable positions and motions systematically by an additive constant, but not by a scale factor. (2) The coverage should include enough reference stars to determine the plate scale as accurately as possible. (3) As many stars of the Trapezium cluster as practical should be included, while equally faint stars farther out may be omitted. (4) The formulation of the adjustment equations should be statistically fully rigorous; that is, the variance of the residuals of <u>all</u> the (weighted) observations, taken together, is to be minimized. This means, among other things, that the mathematics used in the adjustment must take into account the

fact that a given star at a given epoch has one position regardless of from whatever plate this position is derived. Taking due account of this latter restraint is equivalent to using the "overlap" method of Eichhorn (1963). It is the imposition of the overlap condition that produces not only higher internal accuracy than otherwise obtainable, but also allows the reference stars to determine more accurate estimates for the plate scales and hence the expansion. This is so because there are not enough reference stars in the region, and therefore, the accuracy of the plate scale which one would obtain if each plate were reduced separately from all others is insufficient for our purposes. By imposing the overlap condition, however, we are able to make full use of plates on which as few as <u>no</u> reference stars appear.

A <u>fully</u> rigorous solution, in the strict sense defined above, would require that every single measurement be preserved separately throughout the reduction, i.e., no averaging of individual settings of the measuring engine, or of separate grating images, could be made. Since strict rigor is laborious to enforce and really unnecessary when the error distribution of such measurements is uncorrelated with any of the parameters being sought, we have relaxed the algorithm to allow the usual practices of averaging direct and reverse measures, pairs of grating images, etc., in the interest of expediency.

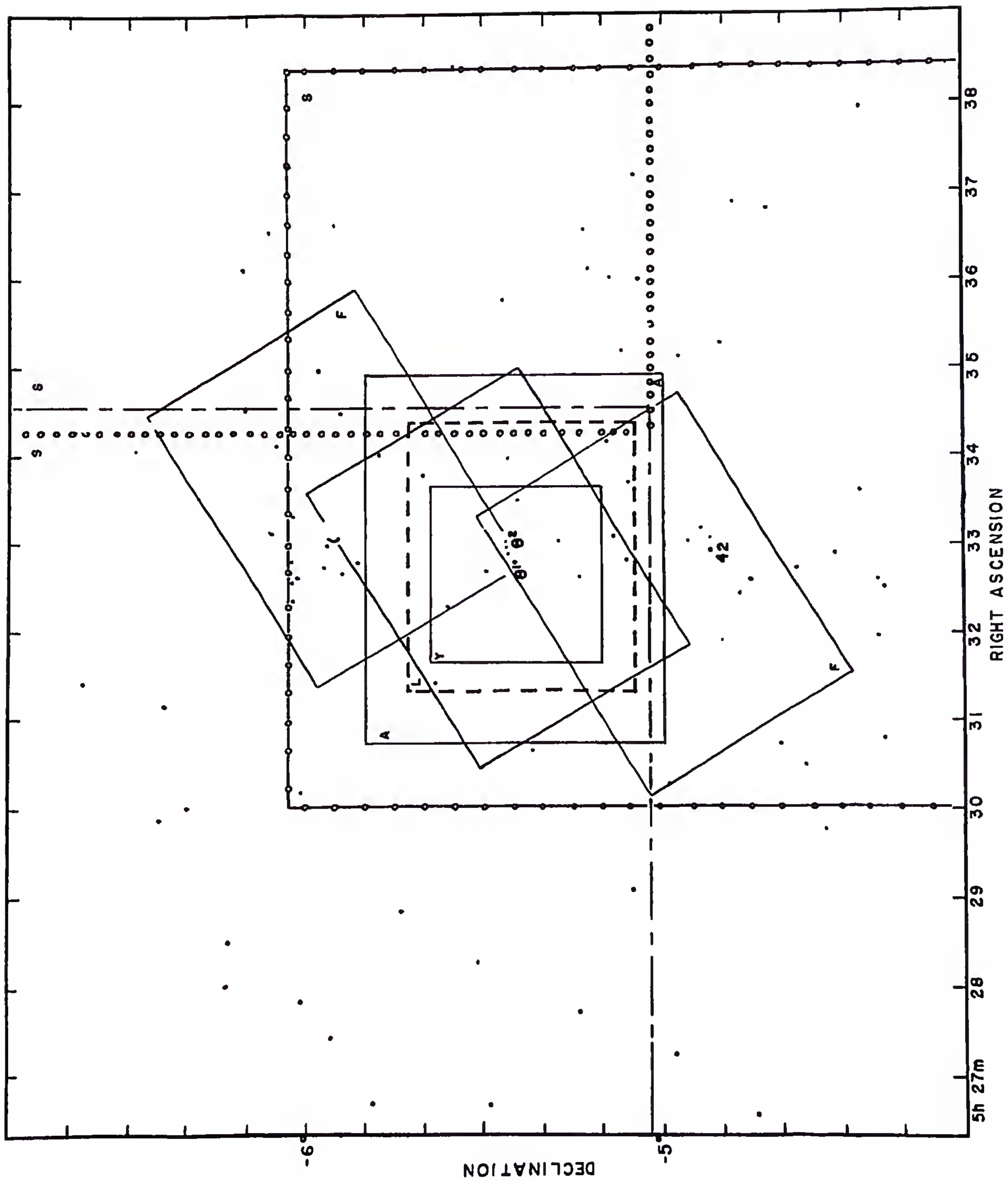
SECTION II
REFERENCE MATERIAL

By reference material we mean the positions and proper motions of the set of reference stars with respect to which the positions and motions of all the program stars are computed. Since the role of the reference material is especially crucial in the present study, it deserves discussion in some detail here.

As usable reference material we may only include positions whose errors are uncorrelated. In practice such positions are obtained from meridian observations, or photographic catalogues sufficiently global that our selection of reference material is confined to a relatively small region. Thus, heliometer or micrometer observations are usable, even when confined to a small region of the sky; so are photographic zone catalogues, if the zone is sufficiently wide, even though such positions are "secondary." On the other hand, we consider a catalogue based on photographs covering only the program region, e.g. the Zô-Sè Catalogue, unusable as reference material because the errors are appreciably correlated through the plate constant variances of the photographs.

FIGURE 2

Location of reference stars and boundaries of the plates, designated as follows: S-San Fernando Astrographic Zone, F-University of South Florida, A-Allegheny and McCormick (coincident), Y-Yerkes. L denotes the region in which we have given the proper motions.


RIGHT ASCENSION
DECLINATION
5h 27m
28
29
30
31
32
33
34
35
36
37
38
-6
-5
42

The region from which the reference material is taken is approximately bounded by $5^h26^m < \alpha < 5^h42^m$, $-6½° < \delta < -4½°$ (orientation 1950). The actual limits were dictated by the bounds of those Astrographic Catalogue plates which happen to include the program region; these extend beyond any of the other plate material.

Source Catalogues. We have taken our reference material from the following sources:

1. FK4 and N30. The position of one of our reference stars, ι Ori, is listed in the FK4. The portions of three additional stars are found in the N30: of θ^1C, θ^2A, and of BD-6° 1255. The N30 positions and proper motions were transformed to the FK4 system according to the tables of Brosche *et al*. (1964). For ι Ori, the mean of the transformed position and that of the FK4 has been used.

2. Boss' General Catalogue (GC): For six of the reference stars, positions are found in only the GC. The positions and motions were taken directly from the SAOC and are therefore on the FK4 system.

3. Yale Zone, Volumes 16, 17: The bulk of the reference stars is listed in the Yale Zone catalogues. Their positions and motions on the FK4 system are taken from the SAOC. When the Yale position is not listed in the SAOC, it was taken from vol. 16 (Barney, 1945), or vol. 17 (Barney, 1945a), and transformed to the FK4 system by adding the corrections (Yale to GC) given in the respective introduction

and the correction (GC to FK4) from Brosche, et al (1964). Volumes 16 and 17 cover the zone −10° to −2°, joining at −6° with vol. 16 being the northern. These two halves had been derived from one single belt of Yale plates and reduced as one zone; the proper motions, however, were obtained from comparison with two different Strasbourg AGK1 zones which join at −6°. A comparison of the proper motions for the same star as it appears in both volumes for 24 stars in common, reveals a systematic difference between the two in the sense μ_δ (vol. 16) = μ_δ (vol. 17) − $0\overset{''}{.}012$/yr. We have therefore added the correction $-0\overset{''}{.}012$/yr to all the μ_δ's from vol. 16. If this is not done, the systematic difference is propagated through the plate constants and a marked spurious contraction of the whole region in δ results.

The epoch of this Yale zone is 1934. The AGK1 positions (epoch 1895) of these stars, used in the formation of the proper motions listed in these Yale volumes, have such large errors ($\sigma \sim 0\overset{''}{.}4$), that the resultant error in the proper motions is $\sim 0\overset{''}{.}01$/yr, corresponding to a positional error of $\sim 0\overset{''}{.}45$ at the epochs of both our earliest and latest plates (1900, 1974). Accordingly, it is very desirable to improve the proper motions of these reference stars with additional reference positions of epoch well before or after 1934.

4. Meyermann's Heliometric Positions: B. Meyermann (1903) measured the positions of 47 bright stars in the

vicinity of the Trapezium in 1903, using the Göttingen heliometer. This series is especially valuable because its early epoch and the relatively high accuracy (~$0\overset{''}{.}25$ in each coordinate) improves the accuracy of the proper motions given in the Yale Catalogue from $0\overset{''}{.}01$/yr to $0\overset{''}{.}007$/yr. Thirty-seven of the Meyermann stars are included in the present program; of these the data for 31 are in the Yale Catalogue; for five, in the GC only. (The remaining star can thus be used as a reference star only for plates taken near the epoch 1903.)

Some 30 years later, in the course of a photographic study of the same region, Meyermann discovered an error in the scale of the heliometer; accordingly, in a subsequent paper (Meyermann, 1938), he gave revised values for the 1903 positions, corrected for the scale error. At the same time, he also transformed the positions to the system of the FK3 by translation and rotation, using parameters derived by a least-squares fit of the heliometer positions to those in the PGC and the "Courvoisier"[1] Catalogues. The data for nine stars in all were available for this transformation. The relations between the systems of the PGC and Courvoisier catalogue to the system of the FK3 were known and used by Meyermann.

---

[1]Meyermann does not identify which of several Courvoisier Catalogues is meant, but in any case he quotes the actual positions.

In making use of these positions, we have first precessed the coordinates from orientation 1900 to 1950 using Newcomb's constants, and transformed the PGC positions quoted by Meyermann to the system of the GC (and eventually to the FK4 system) for the same stars by a least-squares adjustment and applied that transformation to all the Meyermann stars. This amounts to a correction $\Delta\alpha = -\overset{s}{.}040$, arising from a systematic error in the PGC in the amount of $\Delta\alpha = -\overset{s}{.}060$.

5. Southern Reference System (SRS): This is the reference catalogue currently being compiled at the U.S. Naval Observatory from relative meridian circle observations and intended as a counterpart to the AGK3R in the southern hemisphere. It contains the data for six stars in the present program, all of these included also in the Yale Catalogue. The average epoch is 1960, thus affording a valuable improvement in the accuracy of their proper motions.

Preliminary positions for these stars were made available prior to publication of the completed catalogue through the courtesy of Dr. J. Schombert of the USNO.

Combination of reference sources. In the case of stars included in more than one of the reference sources, a weighted mean position, proper motion, and epoch has been computed according to the rigorous expressions (eg. Eichhorn, 1974 p. 113). An exception is made in the case

of the three FK4 and N30 stars, these positions not being averaged with those from other sources. The standard deviations characteristic of each source are listed in Table 1.

Double Stars. The Aitken Double Star Catalogue (ADS) was consulted for any entries lying within the range of our program. None of our program stars shows any clear sign of orbital motion from the observations listed in the ADS, so no orbital corrections have been applied. Two uses were made of the ADS material:

1. Doubles closer than 3" but wider than 1" were rejected from the reference list since they might appear variously resolved on some plates and unresolved on others. Some wider doubles (up to 5") were also rejected on the shorter-focus astrographic plates for the same reason. Doubles closer than 1" have all been retained since they are presumably always unresolved. These criteria lead to the rejection of the following reference stars: ι Ori (on AC plates), BD -4.1171, -4.1172, -4.1185, -4.1186 (on AC plates), and -6.1255 (on AC plates).

2. Astrometric data for doubles were utilized in the following special cases: (a) Wide pairs which occur as two distinct reference stars, all of which happen to show zero relative motion, were assigned equal reference proper motion to both members, formed by the mean of the two catalogue values. This amounted in some cases to a substantial

TABLE 1
SUMMARY OF REFERENCE CATALOGUE MATERIAL

Catalogue	$\sigma_{\alpha,s}$	σ_{μ}	t_e
FK4	$0\overset{''}{.}05$	$0\overset{''}{.}004$/yr	1912
N30	.05	.004/yr	1912
Yale (v. 16 or 17)	.14	.010/yr	1934
GC	.15	.010/yr	1900
Meyermann (H)	.25	.007/yr	1903
S.R.S.	.05	.007/yr	1965
Yale (v. 16 & 17)	.10	.007/yr	1934
Yale + GC	.10	.007/yr	1915
Yale + H	.12	.007/yr	1924
Yale + S.R.S.	.05	.003/yr	1960

departure from the original catalogue value, and always yielded a better fit to the plate measurements. This procedure does not, of course, <u>constrain</u> the final proper motions of the components to be equal. The following pairs were treated this way: θ^1C and θ^2A; θ^2A and θ^2B; BD $-6°1233$ and 1234. (b) For θ^1E and θ^1F, the fainter companions to the four principal Trapezium stars, the micrometric observations prior to 1920 are used in the determination of their proper motions, since these two stars occur on no early epoch plates.

<u>Parallaxes</u>. Only two of our program stars have known and measurable parallaxes: ι Ori (π = $0\overset{''}{.}02$) and 45 Ori (π = $0\overset{''}{.}02$). These values are too small to influence the overlap solution, but are probably large enough to establish the stars as foreground objects. Correction for the effects of these parallaxes have been applied to their reference positions.

<u>Magnitudes and colors</u>. In general, we must anticipate that the plate reduction models may include terms involving magnitude and color. Therefore, we need approximate values for them. For this purpose, they need to be known with a relative accuracy of about $0\overset{m}{.}1$. As a criterion for membership in the cluster, however, we require more accurate values. As data we have (1) used the photoelectric photometry of Johnson (1957) and Sharpless (1962) when available, and (2) computed m_V and m_B for the

remaining stars from measurements on the iris photometer of two pairs of blue and yellow plates, each pair taken simultaneously especially for the purpose, and using the Johnson and Sharpless stars as standards. The details and final results of this photometry will be published elsewhere; the colors given here should be considered preliminary.

SECTION III
THE PLATE MATERIAL

General Survey

In this sub-section the whole of the plate material is summarized. The next two sub-sections deal with the USF plates particularly and with the plate measurements carried out in the present program.

The plate material (that is, the x,y coordinates of the images of the program stars as measured on the photographic plates) used in this study is very heterogeneous. This is so partly by necessity and partly by design. There exists no one series of plates, taken at one telescope, having at once all four of these properties: (a) epoch difference of at least 30 years, (b) plate scale of at least (say) $20\mu/''$, (c) inclusion of an adequate number of reference stars, and (d) measurable images of the inner cluster stars (i.e. stars within 5' of the Trapezium). Thus, the Yerkes plates, for example, bear images of only seven reference stars; the Allegheny plates are obscured at the center by over-exposed nebulosity, while on the McCormick plates the central region lacks all but the brightest stars due to the use of a sector wheel. The U.S.F. and Sproul plates avoid all these difficulties, but

have no early epoch counterparts. As for the possibility of repeating the AC type "astrographic camera" photography of this region with modern plates from the same telescopes, this is not feasible in the present instance because the San Fernando plates covering the region had an exceptionally bright limiting magnitude ($\sim 11^m$), while the early epoch "Zô-Sè"plates (of excellent quality) cannot be repeated because the telescope with which they were taken has disappeared. In any case, the focal length of the astrographic cameras is woefully short--about one-third that of a typical long - focus "parallax" instrument such as the McCormick refractor.

It is regrettable that an objective grating was not employed in <u>any</u> of the early epoch photography.

The plate material is summarized in Table 2. Explanatory notes and references are provided in the caption. A list of the plate numbers of the individual plates used, in the notation of their respective observatories, is given in the Appendix (Table A-1). The information concerning the circumstances of the USF photography, since it has not appeared elsewhere, is also listed in the Appendix (Table A-2).

The USF Photography

Telescope

A description of the astrometric reflector of the

TABLE 2
SUMMARY OF PLATE MATERIAL

Obs[a]	Plate Scale (μ/")	Epoch[b]	No. of Plates	No. of Stars[c]	No. of Ref. Stars[d]	Msmt[e]	Source[f]
S F	17	1893,1906	3	130	85	M	1
Z-S	17	1904-1916	5			M	2
Y	93	1905-1912	7	92	7	M	3
A	68	1921	6	70	15	M,A	4
McC	48	1924-1927	22	47	13	A	5
McC	48	1968	22	47	13	A	5
A	68	1970	5	70	15	A,M	4
Sp	53	1974	2	90	14	A	4
USF	50	1974	20	140	35	A,F	4

[a]Name of observatory: S F = San Fernando, Spain; Zô-Sè, also "Zi-Ka-Wei," Shanghai, China; Y = Yerkes; McC = McCormick; A = Allegheny; Sp = Sproul; USF = Observatory of University of South Florida, Tampa.

[b]Epoch: approximate mean value, or range.

[c]No. of stars: number of program stars appearing on at least 2 plates.

[d]No. of ref. stars: number of reference stars with measurable images on at least 2 plates. (The components of θ^1 and θ^2 are counted here as only two ref. stars.)

[e]Msmt: Type of measuring engine--M manual; A automatic (impersonal); F fringe-counting (screw-independent).

[f]Source: source of measurements as follows:

(1) Astrographic Catalogue, San Fernando Section, 4 (-5° zone), p. 50 (plate 1496); and 5 (-6° zone), p. 50 (plates 1490, 3890).

(2) Chevalier (1933). The listing for plate "2" contains many star misidentifications discovered and, where possible, rectified by this author. There are also several typographical errors in the measurements. This photography has no relation to the Zô-Sè equatorial zone, published shortly before in the same series of publications. In general, all of the photography and measurements carried out under the supervision of Chevalier at Zô-Sè were meticulously and ingeniously done so as to yield very high accuracy.

(3) Strand (1958, 1972) for description; the measurements on punch cards provided by Dr. Strand.

Table 2--continued

(4) Present investigation. Measurements made in 1973 and 1974, described below.

(5) Cannell (1970) for a description; the measurements on punched cards provided by Mr. Cannell. Both the early and late epoch plates were measured by him in 1970.

University of South Florida (USF) Observatory has not yet appeared in the literature. The telescope is a 26-inch Tinsley, f/15 "Schmidt-Cassegrain" with optics designed by J. G. Baker. These are a full-aperture corrector plate (not a "reflector-corrector"),an f/5 nearly spherical CERVIT primary, and an approximately spherical secondary in a Cassegrain configuration. This system has also been termed a "Baker-Schmidt." The unobstructed field is $1^{\circ}\!.2$.

Stellar images are sharp and round up to the edge of the field, exhibiting no trace of coma.

As is the case with systems of the Cassegrain type, image quality is critically sensitive to the collimation of the elements. With the USF telescope, proper collimation is achieved only with difficulty; moreover, the mirror supports are such that collimation cannot be maintained for more than a few weeks at a time. Consequently, the conditions under which the plates used for this investigation were obtained cannot with certainty be duplicated at a later time, and such instrumental plate constants which might otherwise have remained constant over time cannot be assumed to have done so here.

The USF Observatory is located within an urban area and suffers from a sky brightness that sets the limiting magnitude at about 14^m on yellow plates.

Coverage

The format of the USF plates is 5 x 7 inches. Centered at the Trapezium, this format encompasses only about one dozen reference stars. In order to utilize more reference stars we must extend the field by overlapping adjacent plates. The overlap pattern chosen, shown in Figure 2, was dictated by both the distribution of the reference stars--which is far from uniform--and limitations of economy. It is seen that there are very few reference stars east and especially west of the cluster--a circular area one full degree in diameter centered about 1/2° west of the Trapezium contains but two reference stars. The distribution of plates is center-to-corner, with the Trapezium appearing on nearly all the plates; the region covered is roughly co-extensive with the "Sword" of Orion, encompassing a total of 39 reference stars. Sixteen of these plates were used in this study.

Exposures

Photography of the Orion Trapezium cluster presents several challenges. First, of course, is the nebulosity itself. On the plate, the background light of the nebula blots out the fainter stars imbedded in it, while

condensations of the nebula adjacent to stellar images can shift the apparent photo-centers of those images. Second, the magnitude range of the program stars is at least seven magnitudes. Within the vicinity of the Trapezium especially, bright and faint stars are closely intermingled, precluding the use of a sector wheel. An objective grating can only be used with great care, lest the multitude of images of the different stars impinge upon one another.

We have sought to circumvent these difficulties with the following:

1. Choice of filter and emulsion. All exposures were made on 103a-G emulsion, taken through a Corning 3-69 filter. The resultant combination isolates a 600 Å-wide region free of major nebular emission lines. The suitability of this combination is revealed in Figure 1, in which the nebulosity is clearly suppressed.

The filter can be expected to introduce some distortion of the field. In order to counteract this, we position the filter immediately in front of the plate, according to standard practice, the spacing being 3mm. In addition, we have taken the unusual precaution of calibrating the filter, as described below under "Measurement Processing." To facilitate in applying this calibration, a reference mark is scribed on the filter so as to be silhoueted on plates exposed through it.

2. Objective grating. A coarse 26-bar objective grating was constructed and employed in some of the exposures. The direction of the bars was rotated until the diffraction patterns of the Trapezium stars all avoided one another--this proving a most delicate adjustment. The grating constants were found by experiment to be $4^{m}.4$, $4^{m}.7$, $5^{m}.0$, $6^{m}.3$, respectively for the first-through 4th-order images. (In retrospect, it seems probable that a more evenly graded set of constants would have been more useful.)

3. Multiple exposures. As an alternative to the use of the grating, some plates were taken with both a long and a short exposure. An x,y translation in the focal plane of less than 5 mm was found such that the images of one exposure would not fall on any of those of the other. Exposure times of 10^{min} and 1^{min} were usually made, separated in time by only a few seconds while the plate holder was advanced in the camera by the pre-set distance. Some of the plates taken with the grating were also given two exposures.

Photographs were taken on either side of the pier, usually at less than 30 min. from the meridian but occasionally more. Exposure times were usually 9 minutes, reaching $13\frac{1}{2}^{m}$. Particulars are recorded in Table A-2.

All plates were kept horizontal during developing and drying to reduce emulsion shifts. For the same reason, washing time was reduced to a bare minimum even at the expense of incurring cosmetic blemishes, while drying time

was reduced by using a "drying box" with circulating warm air.

Measuring the Plates

General Consideration

As noted above, measurements for some of the plates used in the present study were already available. The other plates were measured in the course of this investigation. The latter are 20 USF plates, 12 Allegheny plates, and two Sproul plates.

Ideally, one of the automatic measuring engines, such as the SAMM (Strand Automatic Measuring Machine) of the U.S. Naval Observatory, would have been used for the measurement of all plates because of both its impersonal centering action and much greater speed. In practice, we found it is not always possible to use such a machine, for the following reasons. (The limitations referred to here apply specifically to the SAMM, but similar limitations apply as well to the other automatic machines.) The effective scanning area perceived by the SAMM centering mechanism is a circle of 250μ diameter. On the SAMM it is impossible to properly measure an image which extends beyond this circle, or which is not isolated within the circle which happens, for instance, when a part of another image also falls within the circle; or when the background varies measurably across the circle's area or is darker than

density .5. These limitations are often exceeded in the Orion nebula photography, due to the crowding together of grating images, the presence of the nebulosity itself, or (in the case of some of the USF photography), the presence of sky fogging. The large range of magnitudes encountered in our program also has the result that, on plates taken without a grating, the brighter stars (these include nearly all the reference stars, which must be measured) produce images with diameters larger than the critical 250μ. This phenomenon is especially at its worst on the early Allegheny plates which were taken without the filter required to compensate for the difference in spectral regions for which the chromatic aberration of the objective is corrected and that to which the plates were sensitive.

The plates were inspected visually to judge the degree to which these effects were present and each plate was accordingly assigned to be measured either on the SAMM or on a manual machine. Some plates were measured in both modes--automatic for the smaller images and manual for all the images. A brief account of the respective measuring procedures follows.

Measuring Procedures

The automatic machine. The SAMM of the U.S. Naval Observatory was made available through the courtesy of Dr. P. M. Routly in 1973 and in 1974 and used for the

measurement of all these plates which could be measured on it. This machine has been described elsewhere (Strand, 1971). Measurement sets were made in "direct" and "reverse" orientations and combined by a least-squares adjustment (hereafter called a "rotation") solving for translation, rotation, and a magnitude term, just as is conventionally done for manually made measurements. The residuals (direct minus final) from these adjustments have dispersions of about 1.5 microns.

The calibration corrections given by the USNO for this machine were applied to the measurements made in 1973. It was found that the measurements made in 1974, after a series of adjustments had been made to the SAMM, were better fitted direct to reverse without the corrections, so the corrections were not applied to them. At most, the corrections amount to 2 microns.

The "fringe-counting" machine. Also at the USNO, there is a conventional Mann two-screw measuring machine which has been modified so that the stage travel is measured by Moiré fringe-counters, as in many automatic machines, while the transport and centering remain unchanged (hence this machine is called a "screw-independent" manual machine). About half of the USF plates were measured on this machine; the Allegheny plates are too large to fit on its stage. The temperature at the machine could be measured but not held constant; therefore, measuring was

halted when the temperature varied by more than 1°.5C. A calibration of the machine that had been made at the USNO shows that the corrections are smaller than one micron, therefore, none were applied. Four settings were made on each image in both 0° and 180° orientations, and the two orientations combined as described above. After the fit, the rms error of the effective combined measurements is 1.75μ.

Manual measuring machine. A conventional two-screw Mann engine at USF was used to measure most of the Allegheny plates and a few of the USF plates. The temperature of the machine remained within a 1°C range during operation. Four settings per image were made in each of four orientations: 0°, 90°, 180°, and 270°, thus giving two pairs of direct and reverse measurement sets. Each direct-reverse pair was "rotated" as above, then the two resulting combined sets also rotated one into the other. The dispersion of the residuals after adjustment confirms the value of measuring four orientations: for each initial rotation the rms error is about 2¼ microns, while for the final rotation, it is 1.5 microns. Thus the two-fold increase of information contained in four as opposed to two orientations is completely effective in reducing the mean error of the final combined measured coordinates.

Automatic and manual machines used jointly. The plates measured on both the SAMM and manual machines were

treated as follows. At the SAMM, all measurable images were measured, in direct and reverse positions, and the measurement sets rotated and combined as for the other plates. At the USF machine all the images were measured, and the measurements were likewise rotated as above. Then the combined "manual" set was rotated to fit the corresponding "automatic" set, using as the transformation terms translation, rotation, and scale. The rms error of this adjustment was typically 2μ. Finally, the transformation parameters were applied to the remaining manual measurements, producing a presumably homogeneous set of measurements for all the images. We recognize that this procedure of combining the measurements from two machines is not fully rigorous. This is so because the adjustment parameters themselves contain some error, and this error, which would not be present if the sets were not combined, infects the manually-made measurements. The rigorous procedure would be to segregate the two sets, and during the plate reductions allow separate translation, rotation and scale terms to be associated with each set. Since, however, at least 20 stars common to both the SAMM and the manual measurements, distributed uniformly over the plate, were available in all cases for the adjustment, we feel no errors greater than 1 micron have been introduced by having adopted this procedure.

Preliminary Processing

It will suffice here to give just a brief outline of the subsequent processing of the measurements. We have taken into consideration the following:

Filter calibration. It should be anticipated that a filter may shift the positions of the images as they appear on the plate, since the filter may well depart enough from optical flatness to introduce a measurable and non-uniform distortion of the field. Such a distortion would not appear as a degradation in image quality, nor necessarily be revealed at any step in the reductions. We control the problem by "calibrating" the filters used at the USF Observatory. This is done by placing on the stage of the measuring engine a grid and over it the filter in exactly the same configuration as the photographic plate and filter occur at the telescope, then measuring the coordinates of the grid line intersections with and without the filter in place. The shifts in the coordinates of the grid then presumably duplicate the distortion produced by the filter at the telescope. The corrections are easily applied by measuring the image of the filter reference mark on the plates (see "USF Photography" above) at the same time the star images are measured. The filter "Y" used for the present USF photography proved to require no corrections even as large as 1 micron.

No calibration has been made of the filters used for the other plates in our study, i.e. the McCormick, later Allegheny and Sproul plates. It is recognized that systematic errors of position and motion may remain by failing to have done so, especially if different filters were used for the early and late plates at the same telescope. In any case, the filter used for the early McCormick plates was broken shortly after the plates were taken.

Grating images. Where higher-order diffraction images appear, the mean was taken of each pair of corresponding images and subsequently treated as a single measurement for that star, with its magnitude taken as the magnitude of the star plus the appropriate grating constant. It can be shown that second-order non-linearity of the plate scale over a one-degree field introduces through this practice an error of less than .01μ. The various orders were not averaged together.

The dispersion of the differences between the central image and/or means of the higher-order image pairs is 3.3μ for both X and Y for the USF plates and 4.1μ for the Sproul plates.

Multiple exposures. For plates with two (or three) exposures, the average translation $(\Delta\overline{X},\Delta\overline{Y})$ between the first and the second (and third) exposures was found. A linear dependence of $(\Delta\overline{X},\Delta\overline{Y})$ on magnitude, as might arise from a difference in magnitude term between exposures, was

allowed for. The values ($\overline{X}$, $\overline{Y}$) were applied to all the second (and third) exposure images, whether or not a first exposure image was present, to form one homogeneous set of measurements. If a small rotation between the exposures also is present, that rotation will be incorporated into the final meaned coordinates with no detrimental result; however, a systematic rotation of the set of single exposures with respect to the set of means of two exposures will be introduced. This procedure is rigorously valid if only the plate was moved between exposures. If the tangential point was also shifted (as in re-centering the guide star) an error is introduced amounting to 1μ over a 1° field only when the shift exceeds 5 mm.

On two USF plates, the shorter exposures appeared to be somewhat elongated. In these cases, the separate exposures were treated as though they were separate plates. Several McCormick plates also bore two exposures each; these were all treated as separate plates.

Refraction and aberration. Since the apparent positions of the reference stars are altered by refraction and aberration just as are the positions of the other program stars, it is only the differential values of these effects, across our 2° region, which need concern us. Moreover, since our plate reduction models include linear and quadratic terms with coefficients to be determined separately for X and Y, it is only third-order differential refraction

and aberration which remain explicity to be allowed for.

The largest zenith distance at which any of the plates was taken is 45°. At this value the maximum second-order differential refraction is $''\!\!.016$; the third-order differential is less than $''\!\!.001$. Third-order differential abberation is also less than $''\!\!.001$. Consequently, corrections for refraction and aberration have not been applied.

SECTION IV
FORMULATION OF THE PROBLEM

The method used for the computation of positions and motions in this study falls into that general category termed "plate overlap" methods. In presenting our formulation, we will develop it from the context of earlier methods, beginning with the simplest, so that points of departure are more clearly seen.

Differential Plate Measurements

The most direct computation of proper motion is simply the comparison of the coordinates of the same star as it appears on plates taken with the same telescope at different epochs. The differences, after terms in arbitrary translation rotation, and scale have been removed by a least-squares adjustment, yield the motions directly. We may write this formulation as follows. Let (x,y) be the coordinates of a star on any plate, and x_o, y_o be initial coordinates of the star, say the coordinates from one of the plates chosen as a standard, at $t = 0$. For convenience we can write μ_x,μ_y for the components of μ parallel to the axes of x_o, y_o, multiplied by the focal length. Then the coordinates and motions for a star on the m-th plate are

related by observation equations involving a function f_m of parameters (p_m) and epochs t_m for that plate:

$$f_m(x,y,p) \equiv (p_1+p_2x+p_3y)_m = x_o+(\mu t_m+\Delta x) \equiv x_o+r_m. \qquad (1)$$

For brevity, we let f_m stand for the vector of two equations, and μ, x_o, Δx, and r stand respectively for the vectors $(\mu x,\mu y)$, (x_o,y_o), etc. The parameters (p_m) are found by a least-squares adjustment for each of the M plates. The term in parentheses on the right side represents the "residual" for each star; it contains a random error component and a component due to the proper motion. The computation of the μ from the M residuals can be done rigorously by writing

$$\mu t_m = r_m \qquad (2)$$

and solving for μ by a least-squares adjustment. It is more customary to combine the material in pairs of early and late plates and compute the mean of the individual proper motions, weighted according to the individual time interval. It is to be noted that this will not yield exactly the same value for μ as will the least-squares reduction, unless the ranges of early and late epochs separately are small compared to the whole range of epochs.

The great advantage of this method (apart from its simplicity) is that as long as the plate centers are

approximately the same (a condition usually met in practice), the higher-order terms in the imaging properties of the telescope do not appear. This means that the corresponding errors in those terms do not appear in the errors of the proper motions; and, since such plate constant errors increase rapidly toward the edge of the plate (Eichhorn and Williams, 1963), the proper motions as derived by plate differences are equally accurate out to the edge of the field, at least in comparison with proper motions derived through classical plate reduction. (Of course, one may formulate the problem this way even when the plates have been obtained with different telescopes, but then the need for higher-order terms in the observation equations vitiates the main advantage of the method.)

The disadvantage is that the terms in translation, rotation, and scale absorb any net constant proper motion, rotation or expansion of the stellar group itself. An attempt is sometimes made to pin down these terms by imposing external constraints: the change in plate scale, for example, can be monitored by a field of known proper motions, photographed at the epochs of the earliest and latest plates. Again, the rotation might be monitored by trailing some of the plates. The trouble with these devices is that they do not really eliminate the problem. As long as any factors which might give rise to rotation and scale terms, such as differences in refraction, plate tilt, measuring

engine parameters, etc., remain unknown, adjustment parameters for them must still be included in the reduction.

An example of the application of this method is Strand's (1958) study of the Trapezium cluster. As expected, the internal dispersion of proper motions is very small (except as noted above), while his value for the expansion has been subject to question.

Reference Proper Motions

The disadvantages mentioned in the preceding section may be eliminated while retaining the advantages of a differential method, by making use of reference proper motions (not positions) when these are available. Then we have, in the same notation as above, a set of observation equations for the m-th plate,

$$f_m(x,y\ p_m) = (x_o+\mu t_m) + x \tag{3}$$

where now μ_i is known for some of the stars. Again a least-squares adjustment for the p's is made for each of the M plates. Then, for each of the remaining stars on the m-th plate,

$$\mu_m = (f_m(x,y,p)-x_o)/t_m, \tag{4}$$

$$\mu = \overline{\mu}_m$$

In order to take full advantage of the material, one should then treat these derived μ's as known values in (3) and iterate.

This procedure is still subject to the restriction that all plates be taken with the same telescope and with the same plate centers, and that there be a sufficient number of stars in the field with known proper motions to determine the six adjustment constants.

Classical Plate Reduction

We can depart from the requirement that all plates be taken with the same telescope with stable optical properties and with the same plate centers (termed together "projection conditions") by referring the measured coordinates to a system of reference star positions instead of to one another. Let the spherical coordinates of the stars projected onto a plane approximately tangent to the plate center, the conventional "standard coordinates," be (ξ,η). Then the observation equation relating these to the measured coordinates is, following the notation used above,

$$f_m(\xi+\mu_x t_m,\ \eta+\mu_y t_m\{p\}_m) = x_o+\Delta x \tag{5}$$

where f_m is in general some polynomial in ξ, η, and magnitude. The f_m is now complicated by the appearance of terms expressing tilt, coma, radial distortion, etc. (See, for example, König, 1962 and Eichhorn, 1963 for general

discussions.) (Here, we have assumed that ξ and η vary linearly with time; this is an approximation valid for the small changes involved.)

These equations are solved for the $\{p\}_m$ by separate least-squares adjustments for each plate. Then for each of the non-reference stars we have, for each plate the vector equation (where ξ now stands for the vector (ξ,η)),

$$\xi_m = f_m^{-1}(x)$$

from which the proper motions are computed by solving for each star the set of m equations

$$\xi_o + \mu t_m = \xi_m \quad .$$

There are two advantages to the plate reduction method over the differential methods. First, it does not require that the plates have been taken with the same telescope and plate centers. This in itself is the decisive factor when the plate material is unavoidably heterogeneous, as in the present program. Second, the plate reduction yields actual star positions, in addition to the proper motions.

The disadvantage, as stated above, lies in the necessity of including higher terms in the observation equation. The presence of these terms <u>necessarily</u> increases the total error due to all the parameters. Consequently, star positions which are computed from them contain large systematic errors, especially toward the edge of the plate (Eichhorn

& Williams, 1963) and also in the spaces between widely separated reference stars. This type of error ("parameter error") is not revealed by the adjustment residuals of the reference stars, but only by comparing the positions of field stars obtained from several plates with an independent position, or with each other. The more closely duplicated are the circumstances of the plates, the less apparent are these parameter errors.

Another weakness of the conventional plate reduction procedure, implicit in equation (5), is the assumption that only the plate measurements are subject to error. In fact, however, the reference star positions are subject to error, too. For example, a position error of $0''.3$ (not at all atypical for the Yale Catalogue) corresponds at the focal plane of a typical "parallax" telescope, to 15μ--which is far larger than the expected measuring error.

One might therefore be tempted to treat each plate as a separate adjustment. Approximately, this can be accomplished by assigning the x and ξ relative weights according to some assumed σ_x, σ_ξ. All that this accomplishes, however, is to divide each residual into a "star" component and a "measurement" component in the ratio σ_ξ/σ_x, effecting a mainly spurious reduction in the measurement residuals. Instead, it is much more important to enforce the condition that the star position correction for a given star be a single value, not a set of values for each plate on which

that star appears. It is the rigorous expression of this condition which prompted the development of the plate overlap method treated below.

Among the proper motion investigations employing the classical plate reduction method there are two of the Orion Nebula cluster: Meurers and Sandmann (1963) and (for some of the material) Parenago (1954).

Differential Overlap

We pause to remark here that the overlap condition can be applied in a differential formulation as well.

In the notation used above, we have for the 1st star on the first and m-th plates

$$f_1(x_1,y_1,\{p\}_1) \qquad t_1\Delta\mu_1 = x_1 + t_1\hat{\mu}_1 + \Delta x \qquad (6a)$$

$$f_m(x_1,y_1,\{p_m\}) + t_m\Delta\mu_1 = x_1 + t_m\hat{\mu}_1 + \Delta x$$

$$t_m\Delta\mu_1 = 0 + t_m\Delta\mu_1 \qquad (6b)$$

and for the i-th star on the m-th plate

$$f_m(x_i,y_i,\{p\}_m) + t_m\Delta\mu_i = x_i + t_m\mu_i + \Delta x_i$$

$$t_m\Delta\mu_i = 0 + t_m\Delta\mu_i$$

Again we compute the $\{p\}_m$ and $\Delta\mu_i$ for the reference stars by a least-squares adjustment and by successive iterations, the μ's of the field stars. In forming the normal equations, the conditions (6b) are weighted with respect to the

observations (6a) by a factor $\sigma^2(x)/\sigma^2(\mu)$ chosen a priori. The matrix of the normal equations can be reduced to dimension equal to the number of $\Delta\mu$, as described in the section "plate overlap." The $\Delta\mu$ will be distributed normally with mean zero and dispersion = $\sigma(\mu)/\sigma(x)$ times the dispersion in Δx.

Of all the formulations, this should give the most accurate proper motions; but to be applicable, the plate material must be homogeneous and there must also be an adequate number of stars with known proper motion within the field of view. These conditions are not satisfied in the present program.

To the author's knowledge this formulation has never before appeared in the literature.

Plate Overlap Method

The Observation Equations

We now write the observation equations in their most general form. In order to illustrate the pattern more clearly, we give here the equations for a reference star (the first star) and a field star (the i-th star) as they occur on the first and m-th plates.

$$
\begin{array}{llll}
f_1(\hat{\xi}_1+t_1\hat{\mu}_1,\{p\}_1 & + F_1\Delta\xi_1+F_1t_1\Delta\mu_1 & =(x_1+\Delta x_1)F_1 & (7a) \\
f\ (\hat{\xi}_i+t_1\hat{\mu}_1,\{p\}_1 & + F_1\Delta\xi_i+t_1\Delta\mu_1 & =(x_i+\Delta x_i)F_1 & (7b) \\
f_m(\hat{\xi}_1+t_m\hat{\mu}_{1x},\{p\}_m & + F_m\Delta\xi_1+F_mt_m\Delta\mu_1 & =(x_1+\Delta x_1)F_m & \\
f_m(\hat{\xi}_i+t_m\hat{\mu}_{ix},\{p\}_m & F_m\Delta\xi_i+t_m\Delta\mu_i & =(x_i+\Delta x_i)F_m & \\
 & + \quad \Delta\xi_1 & = 0+\Delta\xi_1 & (7c) \\
 & \bar{t}\Delta\mu_1 & = 0+\bar{t}\Delta\mu_1 & (7d)
\end{array}
$$

where for brevity $f(\xi,\{p\})$ stands for two functions, i.e., is a 1 x 2 matrix, and $x,\mu,\xi,\{p\}$ stand for the vectors (x,y), (μ_x,μ_y), (ξ,η), $(p_1,p_2...)$ respectively. F_m is the reciprocal of the assumed focal length in the same units as the x, of the telescope with which the m-th plate was taken. (While the focal lengths are, in effect, among the parameters being adjusted, only very approximate values are needed for this role and they need not be revised in subsequent iterations.) The ξ and μ are in units of radians and radians/yr., respectively. Carets denote the values used in the initial approximation.

The tm are the plate epochs in years now measured from the reference catalogue epoch, in the case of reference stars, and from some arbitrary epoch in the case of field stars. We have incorporated the approximation $x(t) = \frac{\partial x}{\partial \varepsilon}\ \mu t = F\mu_x t,\ f\mu_y t$ into the terms involving μ.

In the right hand colum appear the observed quantities. In the equations (7c, 7d) representing the reference

positions and motions, the observed quantities are the departures of the initial values of the reference position or motions from the assumed values; these are equal to zero. The Δ appearing in the right column symbolize the errors of the observations; on the left side the Δ are unknowns. The "$\Delta\mu$" of the field stars are, ordinarily, the μ themselves. Of course there must be at least two observations at sufficiently separated t_m for each field star whose μ is to be determined. If this condition is not met for a particular star, we merely eliminate its μ from the equations.

In order to handle the entire set of equations by the conventional least-squares algorithm, the equations are scaled so that the scaled Δ are members of one normal distribution with a single standard deviation σ_u. Here this standard deviation is measured in radians, hence the factor F in the equations (7a). The proper motion equations (7c) are each scaled by $\bar{t}$, a time value such that $\bar{t}\sigma_\mu = \sigma_u$. Finally, the non-uniformity of the σ of the various catalogues and of the plate measurements is accounted for in the usual manner by multiplying each equation by the appropriate value $\sigma_u/\sigma(\equiv\sqrt{w})$.

The Normal Equations

The matrix of the resulting system of normal equations is a single large array of dimension equal to the total

number of unknowns--rather than, as in the classical reduction, a set of matrices for each plate of dimension equal only to the number of plate constants per plate. This total number of unknowns is very large for the number of plates and stars involved in a typical proper motion study; the program which is the subject of this paper, involving 88 plates and 130 stars, leads to some 2,000 unknowns. Clearly this is an unmanageably large dimension. We therefore have taken steps to reduce the size of the largest matrix which must be inverted.

First, we may always align the plates in the measuring machine so that the instrumental x-axis is nearly parallel to the ξ axis. Then

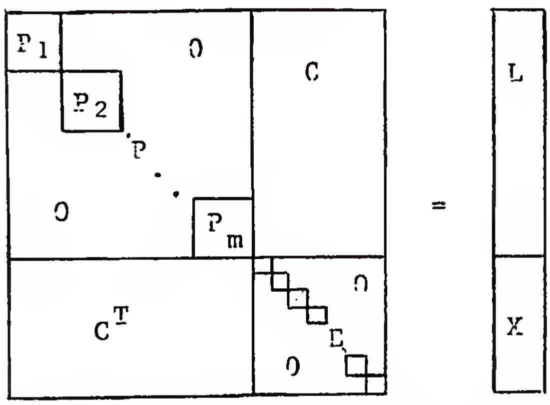
$$\frac{x}{\partial(\xi,\eta,t)} = F(\Delta\xi + t\mu_\xi); \quad \frac{\partial y}{\partial(\xi,\eta,t)} = F(\Delta\eta + t\mu_\eta)$$

Thus the matrix E consists of elements of dimension 2×2 rather than 4×4.

Second, we assume the plate constants p appearing in the equation $f(\xi,\eta,\{p\}) \simeq x$ to be independent of those in $f'(\xi,\eta,\{p'\}) \simeq y$. This means that the normal equations involving the unknowns p, $\Delta\xi$, $\Delta\mu_x$ are decoupled from those in p', $\Delta\eta$, $\Delta\mu_y$ leaving two independent sets of mormals each of half the rank of the original.

The system of normal equations may be represented schematically as follows. Let p_m represent submatrices due

only to coefficients of p (plate constants of m-th plate); E represent the matrix due only to coefficients of star unknowns $(\Delta\xi, \Delta\mu)$: this is a block diagonal of 2 x 2 blocks; C, the matrix of cross-terms, and $V_m(i)$ the vector of plate constant coefficients for the i-th star, on the m-th plate. On the right side, the vector L consists of the homologous vectors of the classical reduction normals, strung together, while the elements of x are given by $x_i = \sum_i W_m F_m X_{mi}$.



In an earlier application of the overlap method, Googe (1967) showed that the system of normals can be reduced in dimension to only the total number of plate constants by "folding," that is, by eliminating E and filling up the entire block containing the P_m. (In that application E was a long diagonal matrix of single elements.) For the task at hand, we instead eliminate the P, folding the matrix into E and producing a solid block of dimension equal only to n x n, n being the number of stars. Since n $\simeq$ 130 (as opposed to about ten times that number of plate constants),

we are left with a matrix that can be easily handled and inverted with present computer capacities.

These reduced matrices, S and T, can be formed in place without the need of auxiliary matrices. They are simply

$$S = E_1 - C_1 P_1^{-1} C_1^T$$

$$T = E_2 - C_2 P_2^{-1} C_2^T$$

where the subscripts merely refer to the homologous counterparts in the matrices of the original normal equations.

It can be seen that the matrix C is composed of strings of vectors such that the i-th row of C is $V_{1,p}(i), V_{2,p}(i), \ldots V_{m,p}(i)$. Thus, the elements of S (and similarly of T) can be written

$$S_{i,j} = w_i(\delta_{ij}) - \sum_{m}^{M} V_m(i) P_m^{-1} V_m(j)$$

with a similar equation for $T_{i,j}$. The Kroenecker delta denotes the original diagonal, $V_m(i)$, is, again, the vector of plate constant coefficients for the m-th plate and i-th star, and w_i are the star weights. For simplicity we have ignored the proper motion components of S. Thus the entire array can be formed from one plate at a time and built up in layers. No more data need be stored at a time than the measurements for a single plate, the inverse P_m^{-1} of the

matrix of normal equations generated by the single plate m, and the arrays S and T with the updated sums as their elements. Only the upper halves of S and T need be maintained, since they are symmetric. Another convenient feature of the formulation is that the data generated by new plates may be added as they become available without the necessity of completely recomputing the already existing S and T.

SECTION V
THE COMPUTATIONS

Partition of the Plate Material and Treatment of the Proper Motion

The plate material naturally divides itself into three groups: an early (~1900 group (A) of wide-field photography; a later (≈1905-1927) group (B) of long-focus photography; and the recent group (1968-1974) of long-focus photography. We have taken advantage of this natural grouping to effect a certain measure of simplification. The procedure adopted is (1) Reduction of the recent group in a single overlap, without regard to the proper motion unknowns, using as initial values for the field stars the positions calculated from plate constants obtained for a subset of these plates reduced using reference stars only; (2) Reduction of the early group as a single overlap, also without regard to the unknown μ; (3) Calculation of a "secondary catalogue" of α, δ, μ_α, μ_δ and means epochs for the field stars, using the results from steps (1) and (2); (4) Reduction of group (B) using this "catalogue" of field stars as reference material of lower weight, in addition to the original catalogue of positions and motions; (5) Calculation of new proper motions of the field stars from the

solutions of steps (1) and (4); and (6) interaction of steps (4) and (5), resulting in the final values of the proper motions. This procedure is justified by the narrowness of the range of epochs in groups (A) and (C), while the 22-year spread of group (B) is adequately compensated for by the use of the approximate proper motions for its field stars.

The role played by the Astrographic Catalogue and Zô-Sè plates is in effect to provide starting values, well determined with regard to scale and introduced systematic errors, for the positions and motions of the field stars appearing on the Yerkes and McCormick plates--these latter plates containing few reference stars. The first-named plates are well suited for this role because they cover a wide area including many reference stars. (See Table 2.) The relatively small scale of these plates is no great disadvantage here since high internal accuracy is not essential for the starting values. (We recall that these initial values are treated as reference material, not merely as starting values.)

In contrast, the late-epoch photography (group (C)), being overlapped, contains a sufficient number of reference stars to allow a straightforward reduction without the need for a "secondary" catalogue (but see below, "Stability of the Solution").

Behavior of the Solution

We need not dwell on the routine mechanics of the calculations. We are concerned here with properties of the solution which are only revealed when the calculations are actually made. To the extent that such properties are not completely predictable in advance, we may say that, in effect, they reflect the stability of the solution. It ought to be remarked at this point that there has not previously appeared a plate overlap reduction of the generality which obtains here, wherein the number of unknowns greatly exceeds the number of reference stars as is typically the case in proper motion studies. The following aspects of the behavior require special mention.

Iterations

A solution of the normal equation yields a set of star position corrections, and plate constants obtained from the initial positions. One therefore replaces the initial star positions for the revised values and iterates the solution until all the star position corrections are less than some value. It was found that convergence is quite slow, requiring about seven iterations for convergence to within $0\overset{''}{.}02$ for all the star positions. Convergence is more rapid when fewer field stars are included as unknowns. If no field stars are included, the solution convergences with only two iterations.

Influence of Weighting

It was found that the solution is rather sensitive to relative weights assigned to the several source catalogues. If too high a weight is assigned to a star, the residuals for that star from the various plates on which it appears reveal a consistent error. Thus it was found that the accuracy of positions and motions from the FK4 and N30 is lower than the quoted mean errors for those catalogues.

If too low a weight is assigned to a star, there is little effect if the catalogue value is very close to the true value. If the initial position is incorrect by, say, 0".4, however, the reduction will over-correct its position. Moreover, it can be argued intuitively that if a reference star occurs in a relatively isolated region, the consequence of assigning its position too low a weight is to allow its position to adjust by too far, without the over-adjustment being evident in the residuals. This is expected because the plate constants are freer to absorb an error in a star position if there are no surrounding stars to help constrain the values of the plate constants evaluated in that region. Indeed, it was found by experiment that when the position of one reference star, 15' from its nearest neighbor, was allowed to vary over a range of 0".8, the *residuals* for that star varied by no more than ".1! The implication to be drawn from this experience is that it is preferable

to overestimate rather than underestimate the accuracy of the catalogue positions.

The weight assigned to the plate measurements relative to the catalogue positions also were found to exert a sensitive influence on the solution. We do not know in advance exactly the mean error to be assigned to the plate measurements, especially those taken from the literature. Within reasonable limits (4μ to 8μ), our choice of the measurement mean errors is guided by the conjecture: given several sets of observations each characterized by an independent error distribution with some σ, that set of σ which minimizes the variance of the adjustment made on the observations is the best obtainable estimate of the σ.

<u>Weighting of Field Stars</u>

Since, by definition, the field stars have no a priori positions, the initial values assigned to them have zero weight; that is, the observation equations (7c)

$$\Delta\xi_i \sqrt{w_i} = \Delta\xi_i \sqrt{w_i}$$

corresponding to them reduce to

$$0 = 0.$$

It was found that, in practice, some non-zero weight must in fact be assigned to the field star positions; otherwise the solution is so ill-conditioned that successive

iterations rapidly diverge. This ill-conditioning worsens as the number of field stars surpasses the number of reference stars, but improves with improvement in the initial values.

We have therefore been compelled to assign a weight to the initial values of the field star positions corresponding to 1/10 that of the Yale positions at the same epoch.

Residuals

There is a residual for each star on each plate. The residuals obtained from the final iterations have the following rms errors, calculated plate by plate, shown here for each telescope. The values obtained from all the plates except a few anomalous cases are included in the ranges given.

TABLE 3
RMS ERRORS

Telescope	rms
Yerkes	$0''.09-0''.06$
Allegheny	.09-.07
McCormick	.08-.06
USF	.09-.04
San Fernando	.40-.25
Zô-Sè	.35-.15

We also compute for each star the average of its residuals for each plate. This mean residual, added to the calculated position correction and the initial position, represents the position for that star as calculated from the final values of all the plate constants. It is this value which is taken as the final calculated position of the star at the corresponding mean epoch of the plates on which it appears.

The Computer Program

The computer program with which these calculations were carried out is written in Fortran IV. It is contained in only about 800 statements and is conveniently modular in structure. It utilizes two tracks of working disk, but requires no tape storage. Taking advantage of the cumulative character of the matrices of normal equations, the program accepts an indefinite number of plates, with an arbitrary (up to the maximum value of n) number of stars per plate, in arbirary order. The number n of stars itself is completely arbitrary, within the limitations of storage capacity. With core storages of 180K, 240K, and 300K, the corresponding maximum values of n are 80, 112, and 135. Were the proper motion explicitly included, the corresponding values of n would be 1/2 of these values. The catalogue of reference stars is treated as an interchangeable data set. Copies of the program on cards are available to interested persons on request.

The Plate Models

By plate model we mean the polynomial terms required to transform the Standard Coordinates of the stars into their (x,y) coordinates on the plate. Each telescope can be expected, in general, to require its own model; finding the proper model for a given telescope is a matter of experiment.

For all plates, the terms whose coefficients are 1, ξ, η, ξ^2, $\xi\eta$ (in the x-equation) and 1, ξ, η, η^2, $\xi\eta$ (in the y-equation) are included, since these terms always arise from the projection geometry. The plate models finally adopted for each telescope include, in addition to the above terms, the following.

TABLE 4
PLATE MODELS

	Terms (and origin)			
Telescope	m (guiding)	xm, ym (coma)	xr^2, yr^2 (radidist)	y^2,x^2 (?)
USF	X			X
Allegheny	X		X	
Yerkes	X	X		X
McCormick	X	X		
Zô-Sè	X	X		
San Fernando	X	X		

No determination of terms involving color could be made, even though such terms are undoubtedly present, because the distribution of colors is too strongly correlated with magnitude and position. Most of the program stars, while not members of the Trapezium cluster proper, are members of the Orion aggregate; consequently, the bluer stars are also brighter. Moreover, nearly all the red stars are concentrated in the near vicinity of the Trapezium. An attempt to model color terms in these circumstances could result in grave, undetectable systematic errors.

It was found that the USF telescope shows neither significant coma nor radial distortion; some plates show a linear magnitude term in x or y. The Allegheny telescope (Thaw refractor) also shows negligible coma, but does exhibit some radial distortion; the earlier plates tend to exhibit larger magnitude terms than do the recent plates.

The Zô-Sè plates show surprisingly small higher-order terms.

The McCormick plates show both linear magnitude terms and large and variable coma, apparently amounting to about 1μ/cm-mag. But the determination of the coma terms for these plates is complicated by several factors: the stars appearing on them, especially away from the center, have a narrow range of magnitude, and they were taken without an objective grating. Moreover there is a strong correlation

between the coma term and the magnitude term in declination. Rather than risk the introduction of unnecessary parameter error, we finally <u>pre-corrected</u> the measurements for the effect of coma, using for the purpose the coma value found earlier by Eichhorn et al (1970) from photography which was better suited than ours for its determination.

SECTION VI
RESULTS

The derived positions and proper motions, given for orientation 1950, are listed in Table 5. The right ascensions and declinations are given in decimals of degrees as the best compromise between convenience of computation and of identification. All the positions are given for epoch 1974.0, this being close to the actual epoch of the recent plates. The weighted mean epoch of the recent plates is also given for each star. The proper motion accuracies are calculated from

$$\sigma^2\mu = (\sigma_1^2 + \sigma_2^2)/\Delta t \quad ,$$

where σ_1 and σ_2 are the mean errors of position at the mean early and mean recent epochs. These in turn were calculated simply from the average mean errors $\bar{\sigma}$ of the n plates on which the star appeared at each epoch according to

$$\sigma_{1,2} = \bar{\sigma}_{1,2}/\sqrt{n} \ .$$

Absolute Proper Motion of the Trapezium

The average proper motion of the Trapezium Cluster is

$$\mu_\alpha = .\!''010/\mathrm{yr}, \ \mu_\sigma = .\!''001/\mathrm{yr}$$

Part of this motion is due to secular parallax, which amounts to (assuming a distance of 500 parsecs)

$$\mu_\alpha = +.\!''001/\mathrm{yr}, \ \mu_\delta = -.\!''005/\mathrm{yr}$$

The galactic rotation term in proper motion in the direction of the Orion Nebula amounts to less than $0''.0005$/yr.

Velocity Dispersion

The most remarkable result emerging from the proper motions is the very small value of the velocity dispersion. This is given in Table 6 for various sub-groups of stars. Here "σ" denotes dispersion, not mean error.

Table 6

Group	Radius	No. of Members	σ_{μ_α} ("/yr)	σ_{μ_δ} ("/yr)
Trapezium	10"	4	.0010	.0009
θ^1 and θ^2	2'	9	.0013	.0012
Trapezium Cluster	6'	43	.0015	.0014
Nebula Cluster	20'	93[(a)]	.0036	.0026

(a): excludes stars assumed to be foreground objects.

The dispersion in proper motions for the inner members of the cluster is so small as to call into question whether any true motion is detectable at all. In any case it sets a realistic upper limit to the true proper motion errors, since random errors will act to increase the observed dispersion. It is believed that these are the smallest dispersions ever obtained for a group of stars.

Translating the proper motions into tangential velocities at the assumed distance of 500 parsecs, we obtain a velocity

dispersion of 3½ km/sec (in one coordinate) for the Trapezium Cluster, 7¼ km/sec for the larger Nebula Cluster. (It is worth recalling that the velocity dispersion of the Pleiades, which is less than 1/3 the distance of Orion, is about 1½ km/sec.) We suspect that part of the difference in the two figures is spurious, due to the increase of error as one moves away from the region near the plate centers and greatest concentration of stars. It is interesting that Strand obtained an equally small proper motion dispersion for the larger Nebula Cluster, but a much larger dispersion for the Trapezium and the Trapezium Cluster (″008/yr., and ″004/yr., respectively). This is probably due to the fact that on the Yerkes plates the nebula and the brighter stars are overexposed.

Contraction

For each proper motion let the radial and tangential components be μ_R and μ_T, the radial distance R being reckoned from the Trapezium. Let the coordinates of the star measured parallel to α and δ, with the Trapezium at the origin be A, D. A linear expansion (or contraction) can be represented by a coefficient k in units ("/year)/R(degrees). The expansion's contribution to μ_R is an amount μ'_R, such that

$$Rk = \mu'_R \equiv \mu_R + \Delta\mu_R \quad .$$

Then in order to find k, we solve by least squares the equations

$$R_i k + (A/R)_i \bar{\mu}_\alpha + (D/R)_i \bar{\mu}_\sigma = (\mu_R)_i + (\Delta\mu_R)_i$$

where $\overline{\mu}_\alpha$, $\overline{\mu}\sigma$ is the mean proper motion of the group. If μ_α, μ_σ are not known, a priori and if the distribution of stars is non-uniform, they also must be treated as unknowns.

The value of k determined from our proper motion in the Nebula Cluster, excluding the high proper motion star is

$$k = (\overset{''}{.}009 \pm .002/\mathrm{yr})/\mathrm{degree} \quad ;$$

that is, a <u>contraction</u> of the cluster amounting to $\overset{''}{.}003$/year at its edge. This result was quite unexpected and prompted a thorough search for sources of error that might have produced it as an artifact. As a weak check on the result we can solve for contraction in α and in δ separately (such "contractions" having no physical significance considered separately); it is found that the two coefficients are equal within one standard deviation. It must be borne in mind that the uncertainty of this result is greater than indicated by its formal error. This is because the contraction coefficient depends critically upon the proper motions of the stars at the edge of the cluster (which in itself is indeed reflected in the formal uncertainty of the result), while it is the positions and hence proper motions of precisely these stars which are most afflicted (as pointed out above, p. 40) and to an uncertain extent by the plate constants errors. In any event, let us accept our result and examine its consequences.

Translated into linear velocity at the edge of the Gerola-Sofia cloud, the R=4 parsecs, the contraction amounts to 10 km/sec. This is in surprising agreement with the value 12 km/sec obtained by Gerola and Sofia for their model of the

cloud and lends strong support to their interpretation of the cloud molecular line widths.

Quite independent of the contraction determination is the evidence of the velocity dispersion in the Trapezium Cluster. Since that result argues against the existence of turbulent eddies with velocities much higher than 3 km/sec, which seem to be required if the cloud is not contracting, we have a double-barrelled support of the view that the Orion Nebula is relatively quiescent, contracting under gravity, and has not yet entered its most productive era of star formation.

If the cluster is not expanding, the "expansion ages" assigned to it in the post are invalid, and stars already formed in the cluster cannot be assigned a single age on the basis of kinematic considerations. The contraction removes any discrepancy between the kinematic and other age determinations, and allows for a continuous range of ages in the member stars.

TABLE 5
POSITIONS AND PROPER MOTIONS

Brun. No.[a]	Notes[b]	Mag.[c]	C.I.	α [d]	δ	μ_α (″/yr)	μ_δ	Mean Epoch[e]	σ_μ [f]
161	a,b	8.72	.51	82.866191	-5.639595	0.0012	-.0490	1969.17	.0018
193	a	11.4	.43	82.921306	-5.204858	.0204	-.0052	69.70	07
203		9.9	.33	82.931950	-5.414630	.0019	.0018	68.58	05
216		10.4		82.945600	-5.647747	-.0074	.0026	69.31	05
213		12.3	.82	82.946243	-5.504052	-.0025	.0004	72.08	06
211	a,b	10.6	.27	82.948028	-5.229830	.0107	-.0036	70.22	04
220		11.3		82.955142	-5.645330	-.0081	.0007	69.59	05
224	EZ	11.5	.70	82.961494	-5.112708	.0115	-.0009	68.22	11
231	a	11.4	1.03	82.966947	-5.485839	.0068	-.0159	69.56	04
252		10.9	.68	82.989454	-5.400735	-.0033	.0016	70.12	04
259		13.2		82.994736	-5.437763	-.0096	-.0026	74.18	06
265		12.0	1.24	82.999595	-5.438656	-.0122	-.0017	72.54	05
272		12.2	1.16	83.001592	-5.564452	-.0056	-.0013	72.54	05
274		13.2	.80	83.004522	-5.448560	-.0026	-.0026	74.17	07
283		12.0	.90	83.007429	-5.431535	-.0062	.0004	72.54	05
286	a	10.9	.30	83.012179	-5.489955	-.0014	-.0185	70.30	04
302		12.4	1.07	83.027306	-5.505792	-.0068	.0004	72.54	05
328		10.7	.73	83.051083	-5.200909	-.0010	-.0039	70.28	04
334		11.4	1.29	83.051262	-5.439323	-.0116	-.0011	72.55	05
335		12.4	.99	83.051889	-5.477202	-.0063	.0012	72.54	06
342		9.59	.04	83.060375	-5.152976	.0011	-.0047	70.32	04
359		11.0	.48	83.066772	-5.339790	-.0098	-.0081	73.22	05
374		12.6	1.13	83.073828	-5.449908	-.0086	-.0023	73.12	06
388		8.13	.16	83.082082	-5.602749	-.0096	-.0016	69.52	04
405		10.3	.65	83.093333	-5.344455	-.0077	-.0011	73.49	04
430	a,b	10.2	1.03	83.103744	-5.584343	.0056	.0053	70.18	04
442		9.0	- .03	83.116392	-5.538014	-.0097	.0007	69.78	04
443	KM	11.8		83.118632	-5.418859	-.0099	-.0006	69.79	04
467	KR	13.2	1.0	83.135866	-5.415684	-.0111	-.0010	73.25	05

TABLE 5
POSITIONS AND PROPER MOTIONS

Brun.No.	Notes	Mag.	C.I.	α	δ	μ_α	μ_δ	Mean Epoch	σ_μ
466	KS	10.4	.15	83.136096	-5.452991	-.0078	-.0012	1971.04	.0004
464		10.9	.46	83.136730	-5.117630	-.0030	-.0041	68.57	06
490		10.5	1.22	83.152109	-5.168715	-.0081	-.0056	70.48	04
497		11.0	1.27	83.156398	-5.279063	-.0104	-.0002	73.20	04
510	LL	11.8	1.24	83.159049	-5.453785	-.0106	-.0007	73.12	05
502		7.8	- .12	83.160352	-5.236157	-.0081	-.0022	69.95	04
522	V356	13.3	.96	83.173676	-5.531420	-.0110	-.0025	72.94	06
526		13.4	1.62	83.176120	-5.422430	-.0105	-.0019	73.65	06
530	LP	8.45	.13	83.176765	-5.496391	-.0103	.0017	69.92	04
535	LR	12.6	1.28	83.179418	-5.470021	-.0117	-.0017	73.22	05
536	LQ	12.4	1.42	83.180157	-5.427356	-.0120	-.0013	73.38	05
542	LU	12.7	1.50	83.183515	-5.465559	-.0116	-.0019	73.49	06
S-76		12.7	2.36	83.187912	-5.427137	-.0101	-.0023	74.52	06
S-85		13.1	2.02	83.197429	-5.409109	-.0138	-.0039	74.61	07
580		12.8		83.198943	-5.413962	-.0105	-.0021	74.50	06
585		13.0	2.29	83.200483	-5.413886	-.0136	-.0011	74.44	06
584	θ^1E	11.3		83.201102	-5.417535	-.0120	.0012	74.21	
587	θ^1A	6.7		83.201326	-5.418813	-.0090	-.0020	70.00	04
591		12.4	.84	83.201884	-5.428698	-.0109	-.0043	73.38	06
595	θ^1B	8.0	.20	83.202608	-5.416713	-.0109	-.0018	71.50	05
S-100		13.2	1.40	83.203656	-5.432372	-.0099	-.0007	74.61	07
598	θ^1C	5.13	- .01	83.204027	-5.421185	-.0108	-.0014	70.27	04
603		10.8		83.205101	-5.421804	-.0083	-.0010	74.23	
601		13.3	2.00	83.205311	-5.432664	-.0126	-.0023	74.67	06
604	MR	10.2		83.205999	-5.394079	-.0122	.0005	73.80	05
615	AE	13.2	1.52	83.206987	-5.390282	-.0112	-.0010	73.98	06
612	θ^1D	6.70	.08	83.207292	-5.419412	-.0092	-.0000	70.77	04
608		9.34	.04	83.209659	-5.096714	-.0082	-.0027	68.11	06
622	MT	12.1	2.41	83.210107	-5.410773	-.0100	-.0012	73.25	05
626	MS	13.0	1.00	83.211753	-5.308601	-.0119	.0013	73.24	06

TABLE 5
POSITIONS AND PROPER MOTIONS

Brun.No.	Notes	Mag.	C.I.	α	δ	μ_α	μ_δ	Mean Epoch	σ_μ
627		12.8	2.00	83.211866	-5.408544	-.0108	-.0019	1974.58	.0006
628	MV	12.7	1.48	83.212957	-5.374170	-.0118	.0001	73.01	05
S-124		13.5	0	83.213548	-5.302027	-.0119	-.0040	74.49	08
633		12.4	.70	83.214915	-5.375550	-.0136	-.0008	72.93	05
643	TV	12.5	1.67	83.219434	-5.380596	-.0134	.0012	73.01	05
651		13.3	2.02	83.219698	-5.475617	-.0117	.0006	72.85	06
655		9.70	0	83.221595	-5.393731	-.0108	-.0014	73.76	05
653	MX	9.85	.71	83.222824	-5.185932	-.0107	-.0005	70.59	04
658		12.3	2.17	83.223134	-5.428346	-.0131	.0000	73.01	05
656		10.9	1.08	83.223286	-5.234933	-.0107	-.0002	70.49	04
669		12.8	2.09	83.226282	-5.429716	-.0129	-.0008	74.52	06
676		12.9	1.05	83.227913	-5.372842	-.0120	-.0010	72.67	07
682	θ^2A	5.07	- .05	83.230948	-5.447424	-.0098	-.0009	69.93	04
690	V358	12.2	1.05	83.235303	-5.544520	-.0112	-.0036	69.40	05
693		13.2	.93	83.240166	-5.211389	-.0149	.0013	69.98	11
696		13.0	1.05	83.241464	-5.195069	-.0142	.0010	72.65	06
698		12.0	.95	83.243688	-5.175777	-.0135	-.0002	72.06	05
703	a,NQ	11.7	.94	83.244427	-5.284536	-.0225	.0118	73.13	05
713		11.6	.86	83.244912	-5.491475	-.0109	-.0014	73.19	05
715	AK	12.7	1.50	83.245368	-5.459123	-.0117	-.0012	72.93	05
714	θ^2B	6.38	- .10	83.245549	-5.448173	-.0103	-.0009	70.85	04
708		13.1	1.55	83.246271	-5.216700	-.0134	-.0029	73.31	06
711	a	11.8	.69	83.246783	-5.251978	.0093	.0011	73.04	05
734		9.5	.38	83.254092	-5.470226	-.0120	.0008	70.51	04
744		12.0	1.04	83.260496	-5.579409	-.0104	-.0016	72.76	05
747	NU	6.81	.26	83.265467	-5.298608	-.0105	-.0017	72.43	05
760	V361	8.21	.04	83.266524	-5.452373	-.0090	-.0014	70.40	05
767	NV	10.4	.45	83.266918	-5.583641	-.0096	-.0022	70.51	04
757		13.0	1.24	83.267338	-5.188969	-.0146	-.0018	71.38	07
773		11.8	1.26	83.270955	-5.550952	-.0114	-.0028	69.99	04

TABLE 5
POSITIONS AND PROPER MOTIONS

Brun.No.	Notes	Mag.	C.I.	α	δ	μ_α	μ_δ	Mean Epoch	σ_μ
776		9.31	.02	83.276779	-5.137060	-.0121	.0006	1970.16	.0005
786		9.90	.09	83.283587	-5.236835	-.0129	.0002	70.33	05
820		13.3	.86	83.304461	-5.481572	-.0129	.0048	73.85	06
831	AN	11.3	1.43	83.310689	-5.501191	-.0110	-.0010	73.13	05
823	a	11.1	.70	83.313409	-5.368083	.1560	-.0071	72.99	05
832	a	11.4	.72	83.314030	-5.260362	.0089	-.0354	72.71	05
848	a	12.8	.94	83.321425	-5.568213	.0540	-.0862	72.84	05
870		12.5	.97	83.332822	-5.313655	-.0189	.0012	72.94	06
864		12.1	.98	83.333421	-5.206084	-.0158	-.0000	72.60	06
884	T	10.8	.70	83.346057	-5.507162	-.0094	-.0012	70.43	05
887		12.5	1.14	83.349230	-5.166690	-.0144	-.0001	72.26	07
907	1330	7.10	-.01	83.362017	-5.659195	-.0032	-.0035	70.29	05
920		9.07	-.01	83.379281	-5.405989	-.0102	-.0026	70.52	05
928	a	11.0	1.13	83.383232	-5.697165	.0223	-.0115	72.55	10
958	1333	9.29	.42	83.529579	-5.091669	-.0063	-.0156	68.12	06
980	a,1334	6.50	-.16	83.449519	-5.678235	.0004	-.0068	73.55	17
992	1335	9.14	-.03	83.498918	-5.438749	-.0031	-.0018	70.41	04
1004	b	11.2	.47	83.519490	-5.425824	-.0020	-.0009	69.87	06
1018		10.0	.64	83.544873	-5.506249	-.0036	-.0042	70.31	04
1016	1338	9.14	.28	83.545441	-5.161193	-.0218	-.0073	68.80	05
1019	1337	10.2	.28	83.549226	-5.361078	-.0111	-.0017	69.92	04
1025	933a	11.0	.42	83.565425	-5.143058	.0057	.0067	68.76	06
1032	934a	10.4	.73	83.587665	-5.512365	.0249	.0097	69.18	06
1057		11.0	.70	83.672020	-5.206745	-.0072	.0047	68.11	12

[a] Brun Number: In the absence of a Brun number, the S number in Strand's notation is given preceded by "S".

[b] Notes: The Greek letter, variable name, or BD number (the latter designated by a four digit number with the preceding "-5°." omitted). a denotes a probable foreground star on the basis of μ. b denotes a probably foreground star on the basis of magnitude and color (Walker 1969).

[c] Magnitude and color: magnitudes in tenths denote photometry by Fallon and Fox (unpublished); magnitudes in hundredths, from Johnson (1957) or, for a few stars, from Walker (1969).

[d] Right ascension and declination for orientation 1950, epoch 1974.0.

[e] Mean epoch: the mean epoch of the <u>recent</u> series of photograph.

[f] σ_μ: the formal mean error of the μ as explained in text, $\sigma_\mu(\delta)$ and $\sigma_\mu(\alpha)$ considered equal.

APPENDIX:

DETAILED SUMMARY OF PLATE DATA

TABLE A-1
LIST OF PLATES USED IN THIS INVESTIGATION
(Numbers are in the system of the respective observatories)

Allegheny	
15 572 (1919)	103 394 (1969)
15 573 "	103 512 (1970)
15 684 "	103 522 "
15 685 "	104 157 "
28 668 (1922)	104 295 (1971)
62 816 "	

McCormick	
17 148 (1924)	77 407 (all epochs 1968)
196 "	408
228 "	580
229 "	581
865 (1925)	624
866 "	625
966 "	650
967 "	651
18 905 "	698
906 "	699
922 "	726
923 "	727
19 480 (1926)	744
481 "	745
564 "	812
565 "	813
20 573 "	814
742 "	815
743 "	844
829 "	845
21 237 "	866
238 "	867

Yerkes	
0 4 (1905)	0 18 (1906)
0 7 (1905)	0 30 (1907)
012 (1905)	0 52 (1908)
013 (1905)	0432 (1922)

Zô-Sè	
"Neb. d'Orion"[a] 1 (1902.913)	4 (1914.120)

TABLE A-1--continued

Zô-Sè--continued	
2 (1910.110)	5 (1916.069)
3 (1911.056)	
San Fernando	
1490 (1893)	
1496 (1893)	
3890 (1906)	
University of South Florida	
13[b] (all epochs 1974)	45
15	55
16[b]	57[b]
22	60
28[b]	63[b]
29[b]	65[b]
44	
Sproul	
T346[b]	

[a]The identification given in Chevalier (1933). The times are given to three places to aid in identification.

[b]Treated as two plates.

TABLE A-2
DATA FOR USF OBSERVATORY PLATES

Serial No.	Date	Grating	Exp.	Center[a]	HA
13[a]	9 Mar 1974.189	-	7 min.	1	- 3 min.
			2		+ 5
15	9 Mar 1974.189	-	2	1	+156
16[a]	10 Mar 1974.192	-	12	1	+ 66
			2		+ 67
22	14 Mar 1974.203	-	9	1	+103
28[a]	24 Mar 1974.230	2	9	1	+120
			1.5		+125
29[a]	24 Mar 1974.230	-	9	1	+140
			1.7		+146
44	17 Sep 1974.710	-	9	1	- 93
45	29 Sep 1974.743	-	8	1	- 35
55	17 Nov 1974.876	-	1.5	2	- 6
			4.5		- 3
57[a]	17 Nov 1974.876	-	5	3	+ 60
			1		+ 66
60	18 Nov 1974.879	-	1	2	- 12
			6		- 7
63[a]	20 Nov 1974.885	2	9	3	- 4
			1		+ 1
65[a]	20 Nov 1974.885	2	4.5	1	+ 78
			1		+ 81

[a]Plate centers: 1 α_c = 5 h. 48 min., δ_c = $-5°$, 24' (Trapazium)

2 30' N, 1 min. W of Trapezium
3 30' S, 1 min. E of Trapezium

[b]Separate exposures reduced as separate plates.

LIST OF REFERENCES

Ambartsumian, V. (1955). Observatory 75, 72.

Baade, W. and Minkowski, R. (1931). Astrophys. J. 86, 119.

Barney, I. (1945). Catalogue of the Positions and Proper Motions of 8248 Stars . . . Between dec's. -6° and -10°. Trans. Astron. Obs. Yale Univ. 16.

Barney, I. (1945a). Catalogue of the Positions and Proper Motions of 8108 Stars . . . Between -2° and -6°. Trans. Astron. Obs. Yale Univ. 17.

Brosche, P., Nowacki, H., and Strobel, W. (1964). Veröff des Astron. Rechen-Institute Heidelberg Nr. 15.

Cannell, W. D. (1970). M.A. thesis, University of Virginia.

Chevalier, S. (1933). Ann. Astron. de l'Obs. Zô-Sè (=Zi-ka-Wei) 18, pt. 1, 16.

Duboshin, G., Ribacof, A., Kalinina, E., and Kholopov, P. (1971). Soob. Gos. Astron. Inst. Sternberg no. 175.

Eichhorn, H. K. (1963). Applied Optics 2, 17.

Eichhorn, H. K. (1974). Astronomy of Star Positions (Ungar, New York).

Eichhorn, H. K., Googe, W. D., Lukac, C. F., and Murphy, D. K. (1970). Mem. R. Astr. Soc. 73, 125.

Eichhorn, H. K. and Williams, C. A. (1963). Astron. J. 68, 221.

Gerola, H. and Sofia, S. (1975). Astrophys. J. 196, 473.

Googe, W. (1967). Astron. J. 72, 575.

Johnson, H. L. (1957). Astrophys. J. 126, 134.

Kahn, F. D., and Menon, T. K. (1961). Proc. Nat. Acad. Sci., 47, 1712.

König, A. (1962). In Stars and Stellar Systems vol. 2: "Astronomical Techniques," ed. by W. A. Hiltner (U. Chicago Press, Chicago), 461.

Liszt, H. S., Wilson, R. W., Penzias, A. A., Jefforts, K.B., Wannier, P. G., and Solomon, P. M. (1974). Astrophys. J. 190, 557.

van Maanen, A. (1911). Récherches Astronomiques Obs. Utrecht 5.

Menon, T. K. (1963). Astrophys. J. 136, 95.

Meurers, J. and Sandman, H. (1963). Veröff. der Univ.-Sternwarte zu Bonn Nr. 65.

Meyermann, B. (1903). Göttingen Mitt. 12.

Meyermann, B. (1938). Astron. Nacht. 266, 381.

Osvalds, V. (1952). Astron. Nachr. 281, 193.

Parenago, P. P. (1953). Sov. Astron. J. 30, 249.

Parenago, P. P. (1954). Trudi Astron. Inst. Sternberg 25.

Popper, D. M. and Plavec, M. (1975). Preprint.

Sharpless, S. (1952). Astrophys. J. 116, 251.

Sharpless, S. (1962). Astrophys. J. 136, 767 = U. S. Naval Obs. reprint #28.

Sharpless, S. (1966). In Vistas in Astronomy vol 8, ed. A. Beer (Pergammon Press, Oxford), P. 127.

Smart, W. M. (1956). Spherical Astronomy (Cambridge U. Press, Cambridge, England).

Strand, K. Aa. (1958). Astrophys. J. 128, 14.

Strand, K. Aa. (1971). In Conference on Photographic Astrometric Technique, ed. by H. K. Eichhorn (NASA CR-1825, Washington), p. 49.

Strand, K. Aa. (1972). Astrophys. J. 174, 721.

Strand, K. Aa. and Teska, T. (1958). Ann. Dearborn Obs. 7, pt. 3, 67.

Trumpler, R. J. (1931). Pub. Astron. Soc. Pac. 43, 255.

Vaerewyck, E. G. (1972). Bull. Am. Astron. Soc. 4, 5.

Vandervoort, P. O. (1963). Astrophys. J. 138, 294.

Vasilevskis, S. (1962). Astron. J. 67, 699.

Walker, M. (1969). Astrophys. J. 155, 447.

Zuckerman, B., and Evans, N. J. (1974). Astrophys. J. 192, L149.

BIOGRAPHICAL SKETCH

Frederick W. Fallon was born and raised in Silver Spring, Maryland, where he attended the public schools, graduating from Wheaton High School. He took the A.B. degree in Astronomy at Harvard in 1961. Subsequently, he worked for seven years at the U.S. Army Map Service (now Defense Mapping Agency) where he developed their program for deriving geodetic positions from the photography of artificial satellites. He also worked during summers as a student trainee at the National Bureau of Standards at the U.S. Naval Observatory.

Returning to graduate school, he attended the University of South Florida where in 1972 he took the M.A. degree in Astronomy with a thesis on photometric evidence for an atmosphere of Io. This work earned an "Outstanding Graduate Student Research Award" from the University of South Florida chapter of the Sigma Xi.

He is currently on the faculty at the University of South Florida, in Tampa, where he lives quietly. His special areas of interest are astrometry and its application to astrophysical problems, optics, and planetary atmospheres. He is the author of some six papers in the professional literature on these topics. He is a member of the American Astronomical Society and of Sigma Xi.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Sabatino Sofia

Sabatino Sofia, Cochairman
Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Heinrich Eichhorn-von Wurmb, Cochairman
Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Frank B. Wood

Frank B. Wood
Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Stephen T. Gottesman
Associate Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Henri van Rinsvelt
Associate Professor of Astronomy

This dissertation was submitted to the Graduate Faculty of the Department of Astronomy in the College of Arts and Sciences and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December, 1975

Dean, Graduate School